Wave Forces on a Moored Vessel from Numerical Wave Model Results

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ABSTRACT

Predicting the wave exciting forces on a vessel typically involves two separate steps; determining a design wave at the location of interest, and then using a linear description of this wave to determine wave forces on a vessel. A method to directly determine the wave exciting forces on a vessel from a wave model would therefore be a very useful tool. This study reports the development, implementation and verification of such a program. The wave model is a non-linear Boussinesq-type 2D model. In determining the wave exciting forces directly from this model, it was desirable to include the non-linear behaviour from the wave model in the determination of the wave forces, as particularly the lower frequency harmonics can have a significant influence on the response of a moored vessel. The conclusion of the study is that the developed program gives a good approximation of the wave exciting forces on a vessel from a non-linear wave field, including the influence of higher and lower order harmonics in the determination of the wave forces.

1. INTRODUCTION

In numerical modelling, predicting the wave-exciting forces on a vessel typically involves two separate numerical programs, one for determining a design wave at the location of interest (this is also often done by physical modelling), and the other using a linear description of this wave to determine wave-exciting forces on a vessel. However a non-linear wave field includes lower and higher frequency harmonics which may have a significant influence on the response of a vessel.

A combination of the two models, namely a wave penetration model that incorporates a 3D ship model is, as yet, not available. The grid size typically used to define a hydrodynamic model, say for a harbour situation, is far too coarse for what is required in a 3D ship model, and the cost and time that would be required to sufficiently tailor a hydrodynamic model has made this unfeasible.

The objective of the study summarised in this paper was to develop a numerical model, termed 2d2SHIPft, to determine the wave-exciting forces on a vessel directly from the results of a non-linear wave model, including the non-linear effects in the determined forces. Essential to this method is the objective to produce time series of wave-exciting forces that correspond directly with the sequence of waves that pass the location of the vessel; the vessel itself not included in the model.

2. STUDY APPROACH

As a first step, a simplified program 2d2SHIP was developed. 2d2SHIP uses linear potential wave theory to determine the Froude-Kryloff wave-exciting forces on a vessel from a harmonic wave defined by the parameters wave amplitude, wave number, wave direction and a time-dependent parameter. For the lateral modes only, namely sway, roll and yaw, the diffraction force due to the presence of the vessel on the undisturbed wave field is determined and added to the Froude-Kryloff wave-exciting force to find the total wave-exciting force for those modes. 2d2SHIP is analysed for a simplified situation, and the results verified against two validated programs, SEAWAY(1) and DELFRAC(2).

A wave-fitting algorithm is then developed to linearise the non-linear wave field from the wave model TRITON(3). TRITON is a Boussinesq-type 2D hydrodynamic model under development at WL | Delft Hydraulics. Specifically this involves parameterising the elevation and depth-integrated velocity components from TRITON, effectively determining a "characteristic wave" for each time step that summarises the TRITON wave field in a linearised form.

This algorithm is then incorporated into 2d2SHIP in creating the program 2d2SHIPft.
2d2SHIPft is validated for a moored tanker under a number of different wave field scenarios. Previous investigations are referenced in validating the method.

3. NUMERICAL SIMULATION OF FLOATING BODIES USING STRIP THEORY

3.1 Numerical Modelling of a Moored Vessel

The equation of motion of a moored vessel is typically represented by the following equation:

$$(m+a)x'' + bx' + cx = F(t)$$

where;

- a and b are coefficients describing the added mass and hydrodynamic damping
- c is a coefficient describing the hydrostatic restoring forces
- $F(t)$ describes the wave-exciting force and other external forces including those associated with restraining the vessel

This paper concentrates on determining the wave-exciting force component $F(t)$ using strip theory. Strip theory is based on linear potential theory; therefore all viscous effects are ignored. The fluid is assumed to be inviscid, homogeneous, irrotational and incompressible.

Strip theory programs divide the problem of the floating vessel in waves into the two simpler problems, namely calculating the wave-exciting force on a fixed vessel, and calculation of the reaction forces on a vessel which is subject to forced oscillation in still water.

3.2 Determination of Wave-Exciting Forces using Velocity Potential

A vessel is assumed to oscillate in the free surface of an incompressible, inviscid, irrotational fluid. The oscillations are assumed to be simple harmonic. The motion amplitudes and velocities are small enough so that only linear terms of the potential boundary conditions and Bernoulli's equation need be considered.

The hydrodynamic pressures on the surface of the vessel are obtained by integrating the linearised Bernoulli's equation using the known velocity potentials. This provides the hydrodynamic forces and moments.

The velocity potential φ can be split into three parts;

$$\varphi(x,y,t) = \varphi_w + \varphi_d + \varphi_r$$

where;

φ_w = incident undisturbed wave potential

φ_d = diffraction potential of waves about the restrained vessel

φ_r = the radiation potential from the oscillatory vessel in still water.

The first two wave forces result from restraining the vessel in still water, while φ_r results from the forced oscillation of the vessel in still water.

The program 2d2SHIPft calculates the incident wave-exciting forces for the vessel by determining the Froude-Kryloff pressure forces utilising the incident undisturbed wave potential φ_w.

The diffraction forces result from the vessel disturbing the incident wave. The forces where the diffraction component is significant are for the lateral forces, sway, roll and yaw, where the presence of the body has a significant impact on the wave propagation in that direction. For these modes only, the total incident wave-exciting force and moment on the vessel is the summation of the Froude-Kryloff and diffraction components.

In determining the diffraction forces, the required added mass and damping coefficients are referenced from a similar form vessel with a similar depth to draft ratio. The selection of the appropriate coefficients for each strip is a function of the geometrical properties and the determined wave frequency of the incident wave at that strip. These coefficients are combined with the appropriate local accelerations and velocities at each depth interval for that particular strip and summed over the cross-section to give an estimate of the diffraction force for that strip.

4. Numerical Program to Directly Calculate Wave-Exciting forces on a Moored Ship

For a given harmonic wave of specified amplitude, direction and frequency, 2d2SHIP divides the wave period into equal time steps, and determines the Froude-Kryloff wave-exciting force, and diffraction force for the lateral modes only, for all six degrees of freedom at each time step.

The analyses for program 2d2SHIP has been carried out for a 200,000 dwt tanker moored in 22.9m water depth for a wide range of wave frequencies and directions.

The results were then verified against two recognised programs, SEAWAY and DELFRAC. Comparison of the results with these programs showed the following:

- The program 2d2SHIP accurately determines the Froude-Kryloff wave-exciting forces and moments for all directions and frequencies.
- Considering only the Froude-Kryloff forces and moments significantly underestimates the total wave-exciting forces and moments for the lateral modes.
- Correcting the Froude-Kryloff wave-exciting forces and moments for the lateral modes by approximating the diffraction forces provides a good representation of the total wave-exciting forces.

5. DEVELOPMENT OF A LINK BETWEEN WAVE FORCE PROGRAM AND NON-LINEAR WAVE MODEL

Critical to the objective of developing the program 2d2SHIPft was to solve the following three fundamental issues:

1. Determining wave-exciting forces on a vessel directly from a wave model without including the vessel in the model.
2. Determining wave-exciting forces from a non-linear wave model using linear theory.
3. Including the effects of the non-linear wave field in the determined forces.

1 Definition of vessel in TRITON wave model

The vessel is not included in the wave model; rather information is extracted at the location of the vessel by defining a number of output points, termed *sampling points,* in TRITON. The total length that the *sampling points* together cover is called the *sampling length*. At each output point, TRITON provides the elevation and depth integrated velocities at that location for that time step. The determination of the wave parameters require derivatives of the TRITON elevation and velocity both parallel and perpendicular to the vessel centreline, therefore additional *sampling points* are required along and either side of the centreline.

2 Formulation of algorithm to determine linearised wave profile from TRITON wave model

The wave-fitting algorithm must link the description of the TRITON wave field in the parameterised form required by 2d2SHIPft to determine wave-exciting forces.

This is achieved, at each time step, by solving and transforming the linearised Boussinesq-type equations.

Considering only a monochromatic wave moving in the positive direction theta with wave speed c, gives the following two equations:

$$\sin\theta\frac{q}{c}-\cos\theta\frac{r}{c}=0 \qquad\qquad \varsigma-\cos\theta\frac{q}{c}-\sin\theta\frac{r}{c}=0$$

where:

q, r =depth-integrated horizontal velocity components in TRITON x,y axis.

These equations are solved for the specified number of *sampling points* covering the *sampling length* to give estimates of wave celerity and direction. This involves solving for a number of points and a least-squares method is used to solve the solution for the overdetermined system.

2d2SHIPft uses the wave celerity and direction to determine the remaining characteristics that define the “characteristic wave”, namely the wave number, amplitude and phase, thereby summarising the TRITON wave field in a linearised form.

3 Inclusion of non-linearities in determined wave-exciting forces

By defining the *sampling points* to cover the full length of the vessel, the phase can be determined directly at each *sampling point* for the TRITON elevation and wave slope at that point and the determined values of wave number, direction and amplitude. By such increased parameterisation of the wave, the “characteristic wave” profile is no longer necessarily sinusoidal, but in fact is a closer approximation of the actual wave profile.

However there is a further effect on the determined wave-exciting forces. In calculating the dynamic pressure forces, there may be variations in the determined wave number over a wavelength due to the non-linearity in the wave profile. By covering the entire vessel with one “characteristic wave” only, this effect will be averaged out.

A solution is to determine multiple “characteristic waves” of specified *sampling length* that together covers the length of the vessel, thereby determining a wave number at different sections of a wave profile. This selection of the *sampling length* is a fundamental parameter to including the presence of non-linearities from the TRITON wave field in the wave-exciting forces determined by 2d2SHIPft.

6. ANALYSES USING PROGRAM 2D2SHIPft

Three separate wave fields were analysed using 2d2SHIPft; (i) a monochromatic wave field; (ii) a bichromatic wave field and; (iii) Wave spectra.

All analyses were carried out for the same 200,000-dwt tanker “located” in the mid-point of a rectangular TRITON domain with a uniform water depth of 22.7m.

6.1 Monochromatic Wave Field

The initial analyses were carried out for a number of monochromatic waves of varying direction and frequency.

The selection of the frequency and direction was chosen to match the analyses carried out for 2d2SHIP (section 4.0), enabling a direct comparison to be made with the previously validated results. In order to make a comparison with the forces determined by 2d2SHIP, the *sampling length* was selected to cover a significant portion of the wavelength, giving a better representation of a linear wave.

The analyses showed that the non-linearities in the TRITON wave signal at the location of the vessel were not well represented by the “characteristic wave”, and that the shape and amplitude of the wave profile determined by 2d2SHIPft was in fact a good representation of the linear wave profile as input into the TRITON model.

Plots of the six-degree of freedom wave-exciting forces produced by 2d2SHIPft show that the amplitude, shape and phase of the forces determined for all modes are a good representation of the forces produced by the direct method in 2d2SHIP. The effect of the non-linearities in the TRITON model are minimised by the large *sampling length*, and the result shows that they have little effect on the determined forces.

The results confirm the methodology incorporated in 2d2SHIPft for determining the wave-exciting forces on a vessel by approximating a linear wave from a time series of elevations and depth integrated velocities over a length of *sampling points* produced by TRITON.

6.2 Bichromatic Wave Field

In linear theory, a bichromatic wave group is the linear superposition of the two wave components. The result is a harmonic wave group, equally distributed about the mean waterline.

However TRITON is a non-linear model. The presence of non-linear wave components means that the component phase angles are not necessarily independent, and there is interaction between the components, termed non-linear interactions, giving rise to secondary harmonics. This secondary interaction also produces group bounded long waves. The long waves lie in the low frequency range, and generally are of very small amplitude. However they may cause large vessel motions in the horizontal modes as a moored vessel's natural wave period is very long and very little damping exists at low frequencies. It was therefore important in the validity and applicability of 2d2SHIPft that these non-linear effects were included.

A reduced *sampling length* was selected to concentrate the determination of the "characteristic wave" on sections of the wave profile, thereby making the algorithm more sensitive to including the presence of the non-linearities in the TRITON wave model.

A check of the magnitude of the forces was undertaken utilising the vessels transfer function determined and validated using 2d2SHIP. This is achieved by undertaking a spectral analysis, and using the relationship that an output spectral density function, in this case the force spectrum is equal to the product of an input spectral density function, in this case the TRITON wave spectrum and the square of the vessel transfer function (Aalbers (4)).

The bichromatic wave consisted of a 1-metre amplitude 16s linear wave component and a 1-metre amplitude 18s wave component.

A plot of the time series of the wave elevation from TRITON at the mid-ships of the vessel is given in Figure 6.1 for a nominated run number 201, together with the linear bichromatic wave as input into the model. The distinct difference between the linear and non-linear waveforms is apparent.

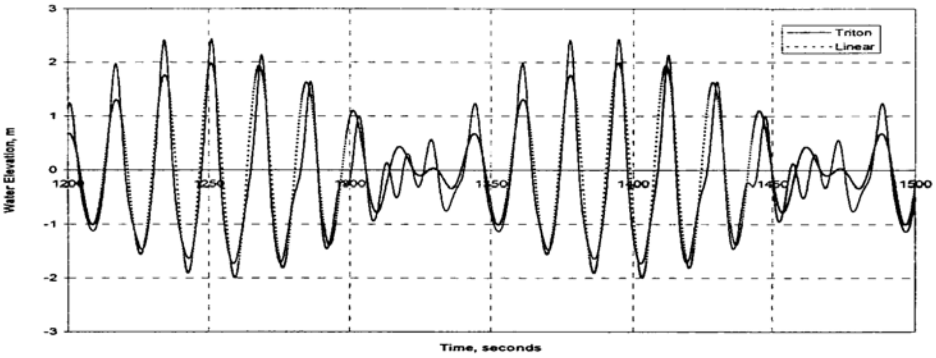

Figure 6.1- Linear and Triton Wave: Run 201

The variation in the individual waves of the wave group can also be seen. The maximum amplitude waves are located in the middle of the wave group, with periods around 17 seconds. Between the wave groups, the individual wave amplitude is at a minimum, with the period reduced to approximately 8 seconds. This shortening of the wave period indicates energy been transferred to the higher frequencies due to non-linear interactions between the two components.

Figure 6.2 shows the "characteristic wave" determined by 2d2SHIPft for run 201. As can be seen, the non-linearities in the determined wave profile are well defined.

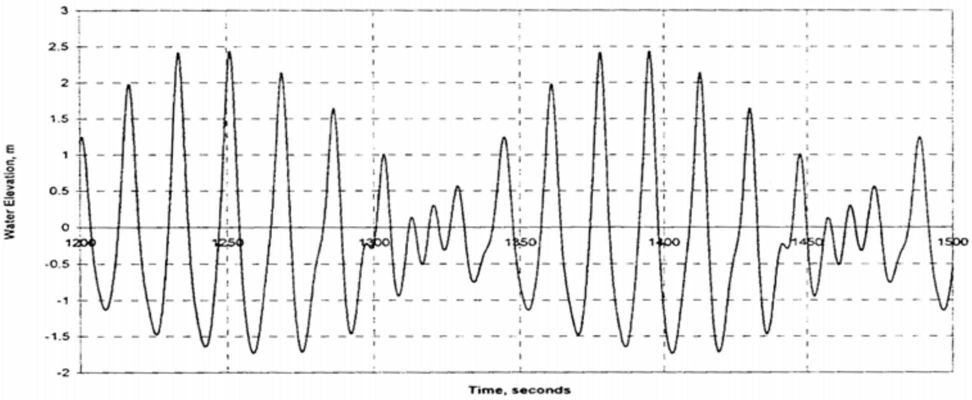

Figure 6.2- 2d2SHIPft "Characteristic Wave": Run 201

Initial verification of the wave exciting forces showed that the forces determined by 2d2SHIPft significantly overestimate the forces determined from a linear bichromatic wave. Furthermore the force profile for all modes was very discontinuous between the wave groups. There should be some difference in the forces predicted by the two methods, as 2d2SHIPft does approximate wave-exciting forces from a non-linear wave field, however the order of magnitude should be equivalent.

The reduced *sampling length* did enable the non-linearity in the wave field to be realised in the wave profile determined by 2d2SHIPft. However this lead to a misrepresentation in the determined wave-exciting forces by incorrectly assuming the wave profile determined from a portion of a non-linear wave profile can be applied over the full length of the vessel. In fact the profile of the wave will be substantially different at another location due to this very non-linearity.

As a result, a new method was incorporated into 2d2SHIPft whereby multiple "characteristic waves" were determined. The vessel was divided into four equal segments with *sampling points* provided along the entire length of the vessel. This enabled a "characteristic wave" to be determined at the different segments of a non-linear wave profile. Covering the length of the vessel also allowed the parameterisation of the TRITON information to be increased, resulting in a wave profile that is no longer necessarily sinusoidal, but is in fact almost an exact replication of the TRITON non-linear wave profile.

To gauge the improvement of the determined wave profile, a plot of the water elevation over the length of the vessel is given in Figure 6.3. Also given is the water elevation profile determined in Run 201 and the actual water elevation in TRITON. Two distinct time steps are provided, one for a peak at the centre of the vessel, and one for a trough.

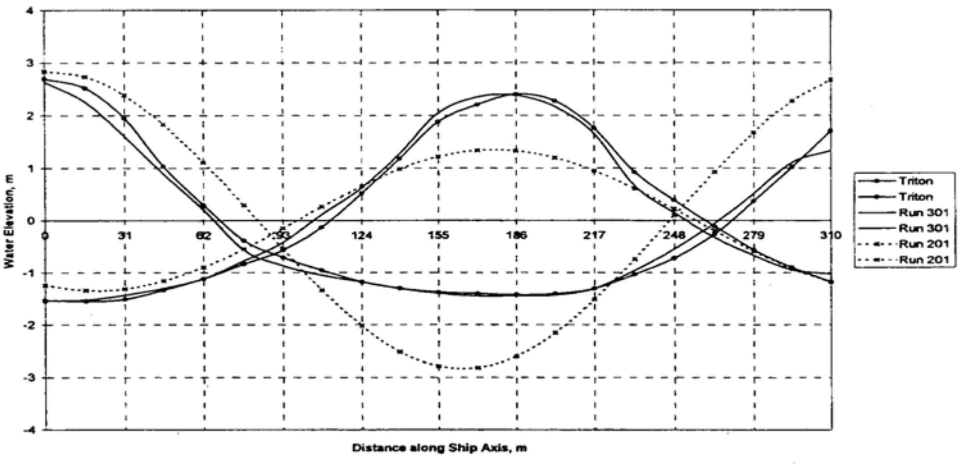

Figure 6.3- 2d2SHIPft "Characteristic Wave over Vessel Profile

The non-linearities over space in the TRITON model can be seen, with the higher amplitude and steeper peaks, and the lower amplitude and flatter troughs. Importantly the wave profile determined for Run 301 very closely matches the TRITON wave profile over the whole wave. Conversely, the error in the original method is very clear, with the monochromatic wave totally missing the wave profile from the TRITON model in space.

Figure 6.4 provides a plot of the six-degree of freedom time series of Froude-Kryloff and total wave-exciting forces determined by 2d2SHIPft.

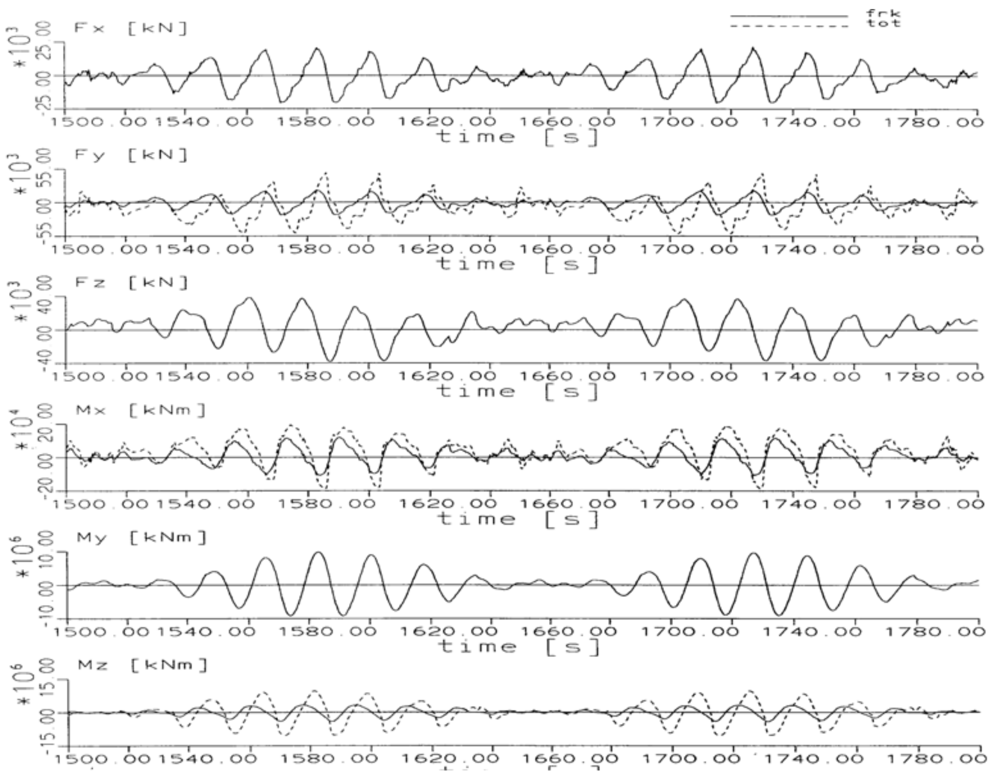

Figure 6.4- 2d2SHIPft Froude-Kryloff and Total Wave-Exciting Forces

Comparison of the wave-exciting forces predicted by the linear and non-linear bichromatic waves leads to the interesting observation that *there may be a difference in wave-exciting forces determined using a linear representation of a wave field, as is conventionally done in numerical mooring analyses, and in using the actual non-linear wave profile from a* *hydrodynamic model to determine the wave-exciting forces, as is done with 2d2SHIPft. This effect should increase as the non-linearity in the wave profile increases.*

To verify that the non-linearities are present and correctly represented in the wave-exciting forces, a spectral analysis was carried out.

The frequency spectrum of the wave signal as input into TRITON shows the energy of the spectrum centred at the dominant peak periods of 18 seconds and 16 seconds, with a very narrow spreading of the energy outside these frequencies. The plot of the TRITON wave elevation frequency spectrum at the location of the centre of the vessel is shown in Figure 6.5.

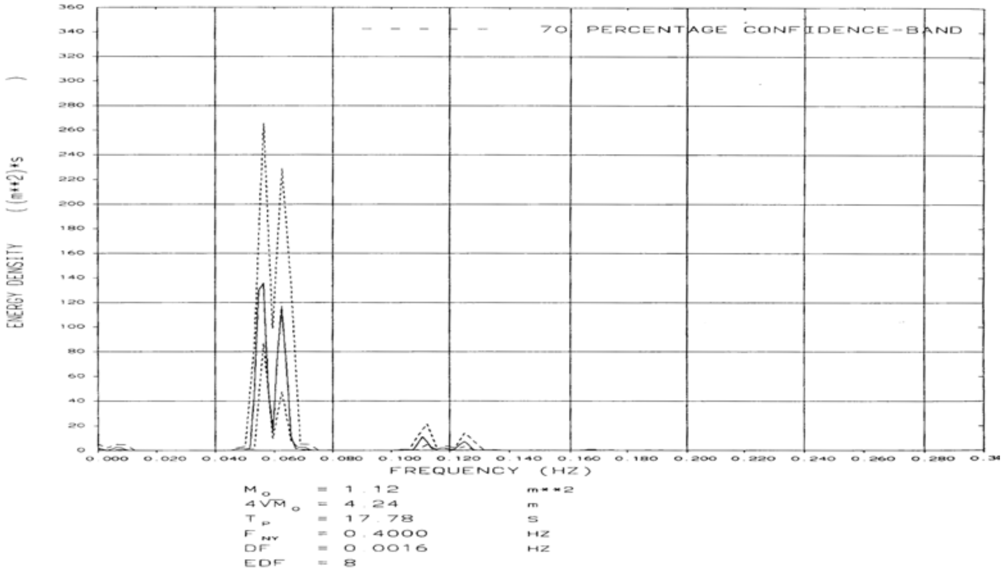

Figure 6.5- Energy Density Spectrum of TRITON Wave Signal

The dominant peak periods around 18 seconds and 16 seconds can be seen. However the energy is now less concentrated around the dominant peaks, and also secondary harmonics are evident. A very low frequency peak corresponds to a period of approximately 144s. This indicates the presence of bound long wave energy attached to the group wave. Furthermore a higher frequency peak can be observed corresponding to the addition of the two frequencies.

The peak period for the input signal was 16 seconds, and at the location of the vessel it is now 17.8s. In propagating to the vessel, energy has been transferred from the higher frequency dominant peak to the lower frequency dominant peak. It is important that this transfer of energy is reflected in 2d2SHIPft. Reference to the vessels response operators (O'Brien (5)) show that the vessel response operator approximately doubles for surge, sway and roll and triples for heave in going from 16 seconds to 18 seconds.

In Figure 6.6, a plot of the frequency spectrum of the "characteristic wave" at the location of the centre of the vessel is given. The dominant peaks and the secondary harmonics are evident, and both their location and magnitude are very similar to the TRITON spectrum, indicating that *the magnitude and spread of the total energy in the TRITON wave field is well approximated by 2d2SHIPft.*

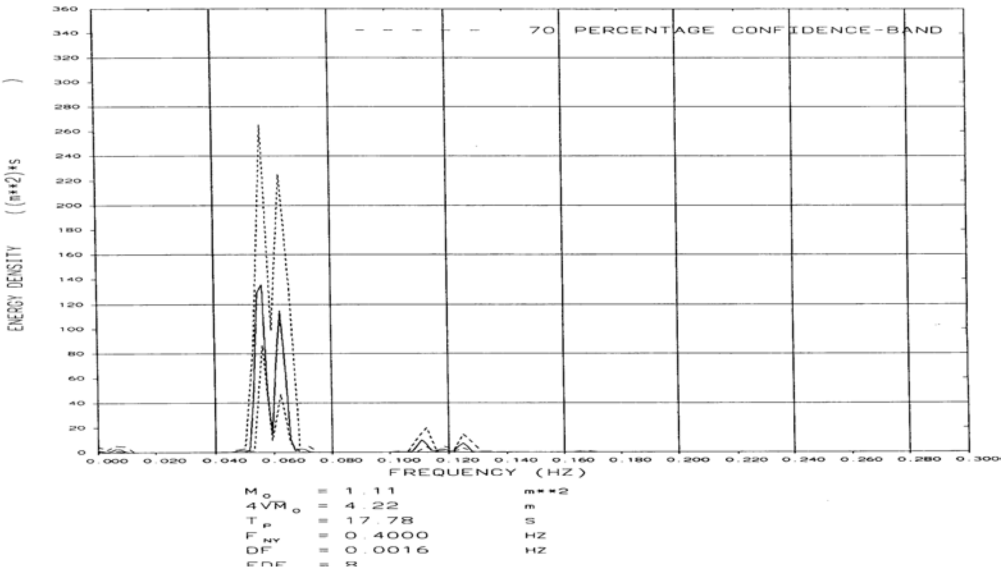

Figure 6.6- Energy Density Spectrum of 2d2SHIP Wave Signal

Similar plots of the force frequency spectrum generated for the six degrees of freedom Froude-Kryloff and total wave-exciting forces were also generated by 2d2SHIPft (O'Brien (5)). The spectral plot for surge is given in Figure 6.7.

These plots show the dominant linear peaks around 16 and 18 seconds, and also the existence of the lower frequency and higher frequency secondary harmonics, indicating that the non-linearities in the TRITON model are reflected in the approximation of the vessel forces by 2d2SHIPft. This was a very valuable result as it indicates that the oscillating second order waves effects are included in the determined forces.

The spectrum of the TRITON input signal showed the magnitude of the energy corresponding to the two dominant peaks to be approximately equal. However Figure 6.7 shows that the energy in the signal is over four times greater for the higher period peak. The vessel response operators at the higher period is approximately double that for the lower period, and given that the frequency based output energy is proportional to the square of a vessel's transfer function, the result is a four times increase in the determined energy for the higher period.

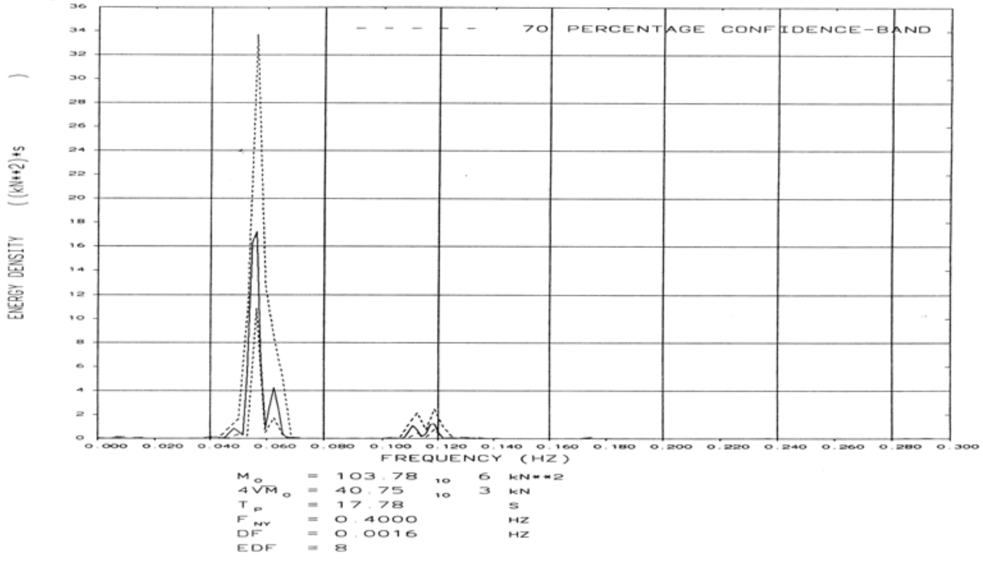

Figure 6.7- Energy Density Spectrum for Surge

This again indicates the sensitivity of the results to correctly including not only the total amount of energy, but just as importantly, the distribution of this energy over frequency. This has been shown to be better handled by determining multiple "characteristic waves".

The magnitude of the wave-exciting force can be verified by dividing the force energy density for a specified frequency by the input wave energy for that frequency, which gives the square of the wave transfer function. This can then be compared to the value determined by 2d2SHIP. A summary of the transfer functions determined by the two methods is given in Table 6.1. The energy density corresponding to the peak period of 17.8 seconds has been used in determining the transfer function from the spectral analysis of the 2d2SHIPft results. The transfer functions determined by 2d2SHIP are for the 17.95 seconds wave as given in O'Brien (5). Also included are the transfer functions determined for Run 201.

Table 6.1 Wave Force Transfer Functions for 2d2SHIPft

	Vessel Transfer Function					
	Surge [kN]	Sway [kN]	Heave [kN]	Roll [kNm]	Pitch [kNm]	Yaw [kNm]
2d2SHIPft - 301	11,450	11,450	22,770	66,000	6,440,000	1,850,000
2d2SHIP	11,180	11,180	27,650	70,860	6,545,000	2,060,000
2d2SHIPft – 201	8,300	8,300	20,000	53,000	5,400,000	2,000,000

In summary, 2d2SHIPft provides a good description of the wave-exciting forces due to a non-linear wave profile, including the contribution from the non-linear components.

6.3 Wave Spectra

Similar analyses was carried out for the following wave spectra:

- a 1.0m Hs 18s Tp JONSWAP spectrum, angle of attack to vessel 225 degrees
- a 1.5m Hs 12s Tp JONSWAP spectrum, angle of attack to vessel 225 degrees

The results (O'Brien(5)) showed that 2d2SHIPft provides a good description of the wave-exciting forces due to a non-linear wave profile for both the 18s and the 12s wave spectrum.

7. CONCLUSIONS

A method has been developed to determine the wave-exciting forces on a vessel directly from the results of a non-linear Boussinesq-type numerical wave model without including the vessel in the model. This method is implemented in program 2d2SHIPft.

The main conclusion is that the method gives a good approximation of the wave-exciting forces on a vessel from a non-linear wave field, including the influence of higher and lower order harmonics in the wave field so important to vessel mooring forces.

Important in the ability of this methodology to accurately include the non-linear wave effects is the suitable selection of a multiple number of wave-fittings, each of a specified sampling length, that together cover the location of the vessel.

A further observation from the study is that determining wave forces on a vessel using a linear representation of the wave field may give different results to that obtained from directly using that non-linear wave profile, with this effect increasing as the non-linearity in the wave profile increases.

8. REFERENCES

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