Estimating Longitudinal and Vertical Centre of Gravity from Vessel Response in Random Waves

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ABSTRACT

Autonomous shipping is a high impact development topic in the maritime navigation industry which is predicated on the development of accurate decision support systems. The accurate prediction of a vessel's centre of gravity is a critical component of ship stability and would tremendously aid real time navigational adjustments to avoid dangerous situations the vessel can encounter. This study was undertaken to estimate the position of the centre of gravity from the full-scale vessel wave induced motions recorded from inertial measurement units. Results showed that the adopted technique can accurately estimate the centre of gravity and calculate the vertical displacements of the ship from the local inertial measurements.

KEY WORDS: Centre of Gravity; Longitudinal Centre of Gravity; Vertical Centre of Gravity; Inertial Measurement Unit; Recursive Least Squares

INTRODUCTION

Having accurate estimation of the centre of gravity (CoG) position is a critical element of full-scale vessel motion analysis and data processing. Errors in the CoG position could lead to unsafe situations from a stability and under keel clearance (UKC) point of view. Different techniques have been proposed to estimate the CoG position from inertial measurement units (IMUs), mostly for aircraft (Johnsen, et al., 2021) (Komendat, 2012) (Angel, 2019) (Chhoeung, et al., 2019) (Linder, et al., 2015) (Al-Rawashdeh, et al., 2014).

This study was looking at the possibility of using recorded motions by IMUs to estimate the CoG of vessels not only for ship stability monitoring, but also for decomposing the measured data and calculating the vertical displacement motions. In this study, the local tangential accelerations and the angular velocities of full-scale vessels, measured by IMU were used to estimate the distance between the measuring point (local IMU position) and the CoG using a recursive least squares method. Subsequently, the longitudinal and vertical centers of gravity (LCG and VCG) are estimated, provided the IMU position from a reference point on board is known, for example: bow, stern, mid etc.

The developed formula is based on kinematic equation (Al-Rawashdeh, et al., 2014) and assumes that CoG is also the centre of rotational motions which is a standard assumption (Lewis, 1989). In the following section, mathematical equations based on the angular velocities and tangential accelerations for estimating CoG position are presented.

THEORETICAL BACKGROUND

When the structure is not restrained, it is free to move. All points of a rigid body will experience the same angular velocities and accelerations; however, the experienced tangential velocities and accelerations of any arbitrary point can be different. Inertial measurement units (IMUs) can measure the rotational and tangential velocities/ accelerations. According to the characteristics of rigid body motion, the experienced rotational motions of any arbitrary point on rigid body would be the same. In other words, the experienced roll, yaw and pitch at any point of the vessel would be the same, but the local linear accelerations are dependent on the distance from centre of gravity. To extract the body displacement from the measured data at any specific point, the distance between the measuring point and CoG is required.

Adopting a flat non-rotating Earth model, the measurements of an IMU located at point P are given by (see Fig. 1):

$$\overrightarrow{A_I} = \overrightarrow{\ddot{R}_I} + \dot{\vec{\Omega}} \times \overrightarrow{R_v} + \overrightarrow{\ddot{R}_v} + 2\vec{\Omega} \times \overrightarrow{\dot{R}_v} + \vec{\Omega} \times \left(\vec{\Omega} \times \overrightarrow{R_v}\right) - \vec{g} \tag{1}$$

where:

$\overrightarrow{A_I}$, inertial acceleration of arbitrary point P measured in body coordinate system ($O_b\ X_b\ Y_b\ Z_b$).

$\overrightarrow{\ddot{R}_I}$, linear acceleration of the origin of the body coordinate system with respect to inertial space ($O_n\ X_n\ Y_n\ Z_n$).

$\overrightarrow{R_v}$, vector from the origin of the body coordinate system (O_b) to point P.

$\vec{\Omega}$, angular velocity of the body system.

$\dot{\vec{\Omega}}$, angular acceleration of the body system.

$\times$, denotes the cross product between two vectors.

$\vec{g}$, gravitational acceleration.

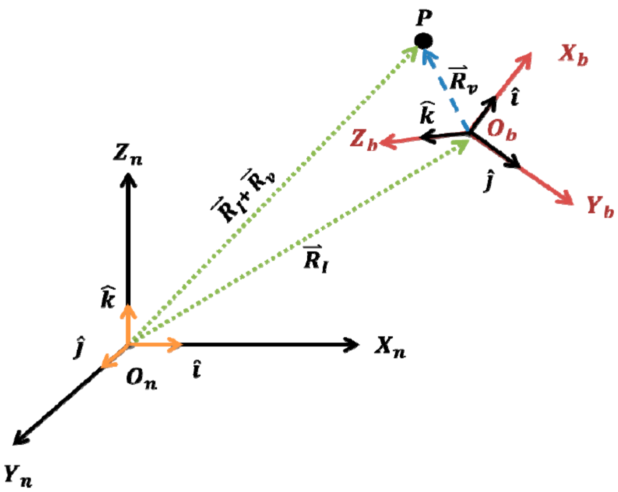

Fig. 1: Inertial $(\boldsymbol{O_n\, X_n\, Y_n\, Z_n})$ and body frames $(\boldsymbol{O_b\, X_b\, Y_b\, Z_b})$

Now, if the CoG is stationary, i.e., the distance between the point P (IMU position) and CoG is constant, then $\ddot{\vec{R_v}} = \dot{\vec{R_v}} = \vec{0}$. Therefore, the measurement at point P is given by:

$$\vec{A_I} = \ddot{\vec{R_I}} + \dot{\vec{\Omega}} \times \vec{R_v} + \vec{\Omega} \times (\vec{\Omega} \times \vec{R_v}) - \vec{g} \qquad (2)$$

which can be written as:

$$\vec{A_I} = \ddot{\vec{R_I}} + B \times \vec{R_v} \qquad (3)$$

By using matrix notation, the accelerometer's measurements at P can be given as:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + B \begin{bmatrix} r_{v_x} \\ r_{v_y} \\ r_{v_z} \end{bmatrix}$$

$$B = \begin{bmatrix} -\Omega_z^2 - \Omega_y^2 & -\dot{\Omega}_z + \Omega_x\Omega_y & \dot{\Omega}_y + \Omega_x\Omega_z \\ \dot{\Omega}_z + \Omega_x\Omega_y & -\Omega_z^2 - \Omega_x^2 & -\dot{\Omega}_x + \Omega_z\Omega_y \\ -\dot{\Omega}_y + \Omega_x\Omega_z & \dot{\Omega}_x + \Omega_z\Omega_y & -\Omega_y^2 - \Omega_x^2 \end{bmatrix} \qquad (4)$$

The above equation, can be rearranges in the form of:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} -\Omega_z^2 - \Omega_y^2 & -\dot{\Omega}_z + \Omega_x\Omega_y & \dot{\Omega}_y + \Omega_x\Omega_z & 1 & 0 & 0 \\ \dot{\Omega}_z + \Omega_x\Omega_y & -\Omega_z^2 - \Omega_x^2 & -\dot{\Omega}_x + \Omega_z\Omega_y & 0 & 1 & 0 \\ -\dot{\Omega}_y + \Omega_x\Omega_z & \dot{\Omega}_x + \Omega_z\Omega_y & -\Omega_y^2 - \Omega_x^2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{v_x} \\ r_{v_y} \\ r_{v_z} \\ a_x \\ a_y \\ a_z \end{bmatrix} \qquad (5)$$

Since the number of equations/matrix row (three equations) is less than the number of variables/matrix column (six variables), there is no unique solution. In this study, a solution was attempted using recursive least squares with a forgetting factor. Using a forgetting factor helps in enhancing the performance when tracking time-varying systems (for example the centre of gravity could change due to fuel consumption or loads' displacements) (Al-Rawashdeh, et al., 2014). The identification problem is solved as follows by rearranging the Eq. 5 in the form of:

$$\vec{A_I} = H\,\vec{\theta} \qquad (6)$$

RESULTS AND DISCUSSION

To test this technique for estimating the centre of gravity of a vessel, both synthetic and real data with known CoG were gathered for algorithm development. Since the practical accelerometers' measurements include noise and are subjected to faults, firstly the proposed method was validated against noiseless simulation datasets. For this purpose, a series of random numbers were generated as linear and rotational accelerations and velocities $(a_x, a_y, a_z, \Omega_x, \Omega_y, \Omega_z)$ at the CoG, then it was assumed that IMU was located at specific location $(r_{v_x}, r_{v_y}, r_{v_z})$ from CoG. With using Eq. 5, the accelerations (A_x, A_y, A_z) at simulated IMU position (P) were calculated. Then the simulated accelerations at simulation point were considered as input to validate the proposed method to compare the estimated location with the simulated values $(r_{v_x}, r_{v_y}, r_{v_z})$. It was found that the method can estimate the IMU position with very great accuracy (sometimes less than 1% error); however, the preliminary investigation results are not presented in this paper.

Next, the technique was validated with actual vessel response measured by an "iHeave"; an inertial motion measurement device developed by OMC International to measure heave, roll and pitch motions (Hibbert, et al., 2013). Fig. 2 shows the latest version of iHeave, which opens up opportunities for data analysis and machine learning techniques for port and ship intelligence by improving access to vessel motion data collection.

For this study, a set of data for a voyage across the Bass Strait measured by an iHeave was selected. The accuracy of the data was investigated previously against DGNSS R8s (Hibbert, 2013).

Fig. 2: iHeave v2.0

The distance between iHeave and CoG was reported as: $r_{v_x} = -23.3\ m$, $r_{v_y} = 0.3\ m$, $r_{v_z} = 23.7\ m$. In the pre-processing stage, the raw measured data was filtered to cancel-out undesirable noises. The noise-free processed data at the iHeave's IMU position was used as input. Fig. 3 and 4 compare the raw and filtered translational accelerations and rotational velocities at this position. In the following, this position (original) will be referred as scenario 1. Then the filtered data at the IMU position was used to estimate the distance between the CoG and the IMU position. Table 1 presents the reported (original) and estimated distance.

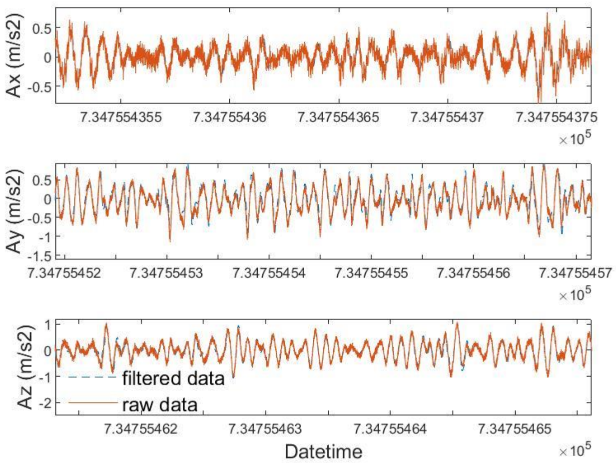

Fig. 3: Comparing raw measured linear accelerations with the processed at IMU position.

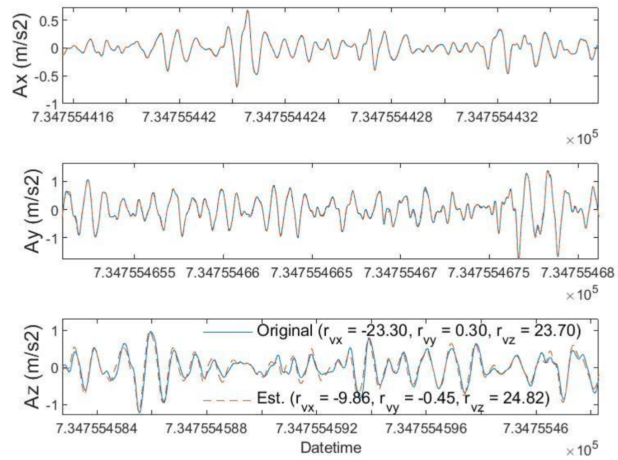

Fig. 5: comparing the accelerations at IMU (original) position and the estimated position.

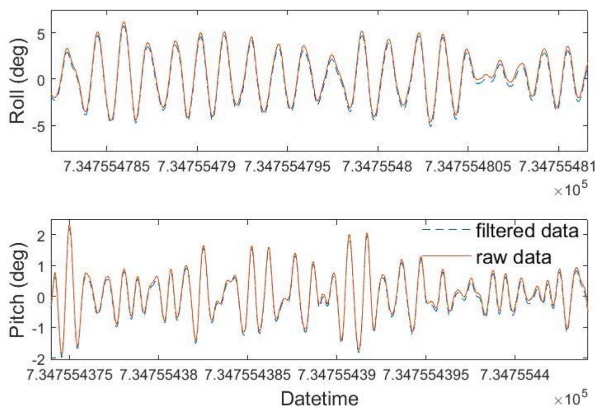

Fig. 4: Comparing raw measured rotations with the processed motions for scenario 1 (Actual position)

Table 1: The iHeave actual (reported) distances vs the estimated values.

	Original value (m)	Estimated value (m)
r_{v_x}	-23.3	-9.86
r_{v_y}	0.3	-0.45
r_{v_z}	23.7	24.82

The estimation was not accurate in longitudinal (x) and lateral (y) directions; however, the error in vertical (z) direction was less than 5%. To investigate more, the accelerations at the original point (IMU position) were compared with the estimated point and partially plotted in Fig. 5. The calculated accelerations in the estimated point are very similar to the accelerations at IMU position. As mentioned before, since the number of equations is less than the number of variables, there is not a unique answer for those set of equations and there is possible that code converges to other points which experienced the same accelerations. Fig. 5 compares the accelerations at the original point (IMU position) and the estimated point. However, those two points are located at different distances from the CoG, they experience almost the same linear accelerations.

THE EFFECT OF IMU POSITION ON THE ESTIMATION

Per the presented result, the solution accuracy for the vertical distance estimation was better than for the other axes. It was theorized that the larger distance from CoG compared to other axes could explain the better estimation. Therefore, to investigate this matter a few other scenarios were investigated using Eq. 5 with the motions at CoG, such that the linear accelerations at any point on the rigid body can be calculated.
In this second scenario, it was assumed a (simulated) iHeave was placed at the same longitudinal and vertical locations, but at the edge of bridge wing (half of the vessel beam) as shown in Table 2. Then, the sensed linear accelerations at the considered position were calculated with Eq. 5, where of course the rotational motions remain the same and used as the input data. Table 2 compares the considered distance values and the estimations. Generally, with increased distance between the measured point and CoG, more linear accelerations will be sensed. It can be seen that the estimated values for new position are more accurate, especially in longitudinal and lateral directions. Fig. 6 shows the linear accelerations at the selected point and the estimated point.

Table 2: The artificial iHeave position vs the estimated position for second scenario

	New position value (m)	Estimated value (m)
r_{v_x}	-23.3	-17.15
r_{v_y}	11.8	12.13
r_{v_z}	23.7	25.16

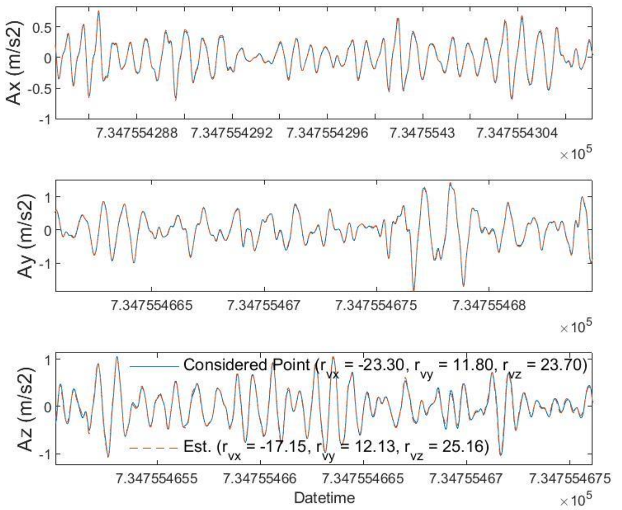

Fig. 6: the comparison of the linear accelerations at considered point and the estimated position for the second scenario.

In the third scenario, the longitudinal position was changed while the lateral and vertical distances were kept the same as for the second scenario (Table 3). According to the results, the order of accuracy for the estimated position in longitudinal direction is almost the same as the previous (error of 26%) but the errors in lateral and vertical directions are smaller. This investigation emphasizes the importance of the IMU position in the accuracy of the estimated values. With comparing the linear accelerations at the estimated position with the considered position, as shown in Fig. 7, very small variations were detected.

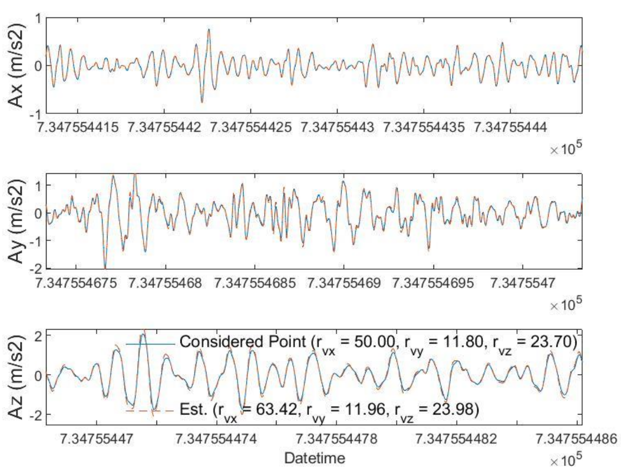

Fig. 7: comparison of the linear accelerations at considered and the estimated positions for the third scenario.

Table 3: The artificial iHeave position vs the estimated position for third scenario

	New position value (m)	Estimated value (m)
r_{v_x}	50	63.42
r_{v_y}	11.8	11.96
r_{v_z}	23.7	23.98

To decompose the local motions (at the IMU position) and calculate the body heave motion (at CoG), accurate distances between the IMU and CoG is required. Therefore, in the following the heave at CoG is calculated based on the actual (reported) and estimated CoG position from IMU and compared.
The RMS between local heave (at iHeave position) and calculated heave at the actual CoG was investigated using Eq. 7 which is 0.2419 m:

$$\sqrt{\frac{\sum_1^N(Heave_{@IMU}-Heave_{@CoG})^2}{N}} = 0.2419\ m \tag{7}$$

It also should be mentioned that heave at CoG is calculated by:

$$Heave_{@CoG} = Heave_{@IMU} - (r_{v_x} * -\sin(Pitch) + r_{v_y} * \cos(Pitch) * \sin(Roll)) + r_{v_z} * (1-(\cos(Pitch) * \cos(Roll))) \tag{8}$$

where all rotational motions are in radian.
Then, the estimated heave at CoG for all three aforementioned scenarios have been compared with actual heave at the reported CoG position. Additionally, RSME for each scenario calculated:

$$RMSE = \sqrt{\frac{\sum_1^N\left(Heave_{@CoG_{estimated}} - Heave_{@CoG_{Actul}}\right)^2}{N}} \tag{9}$$

Scenario One - original IMU position

Comparing the calculated heave at CoG with using the actual (reported) distances and the estimations shows the good accuracy (Fig. 8). According to the results, the RMSE of this scenario is 0.14 m (Fig. 9) which is smaller than the value calculated in Eq. 7. The maximum experienced heave during the transit was 2.4 m. Fig. 9 shows the relative error histogram and RSME value for Scenario one and Fig. 10 plots correlation between the heave at actual CoG and the estimated CoG.

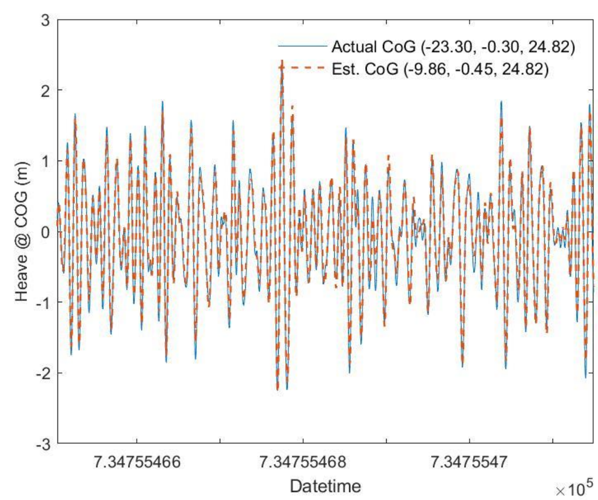

Fig. 8: comparison of calculated heave at CoG with using the actual (reported) distances and the estimated values for the first scenario.

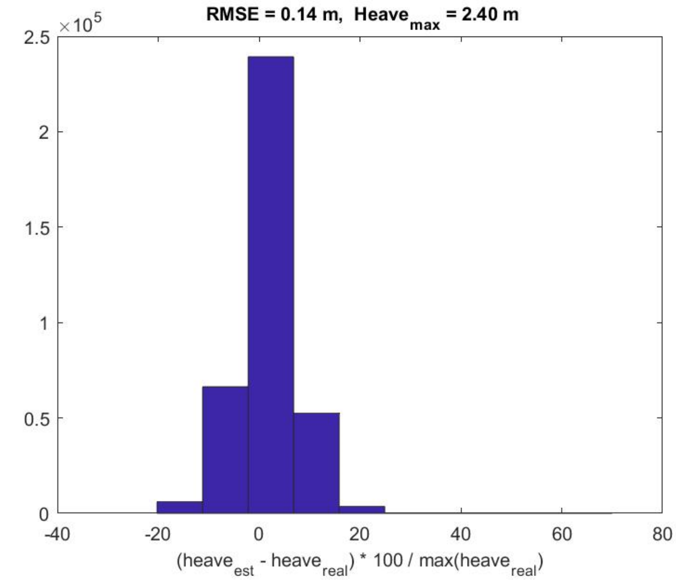

Fig. 9: the relative error and RMSE for the first scenario.

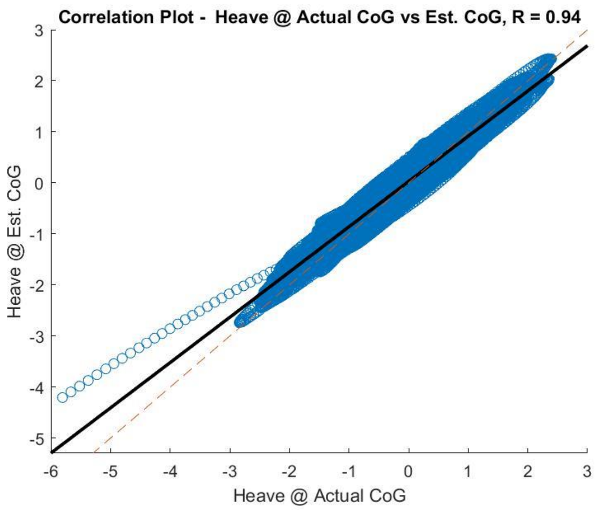

Fig. 10: Correlation between calculated heave using actual distances and the estimations for the first scenario.

Second scenario: IMU at maximum lateral distance (edge of vessel's beam)

The comparison of heave using the estimated values for second scenario is shown in Fig. 11 and the error is presented in Fig. 12. As illustrated, the accuracy of the estimation is improved over scenario 1, which collaborates the hypothesis that estimation of the IMU position can be affected by the relative axial distances. Fig. 13 depicts correlation between the calculated heave at CoG using actual distances and the estimations.

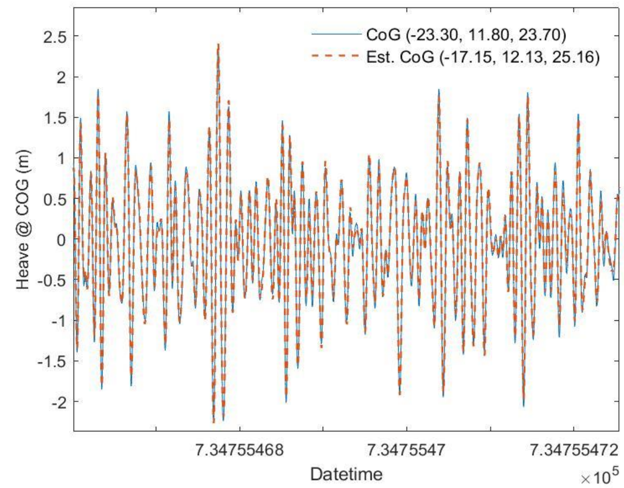

Fig. 11: comparison of calculated heave at CoG using actual/reported with the estimated distances for the second scenario.

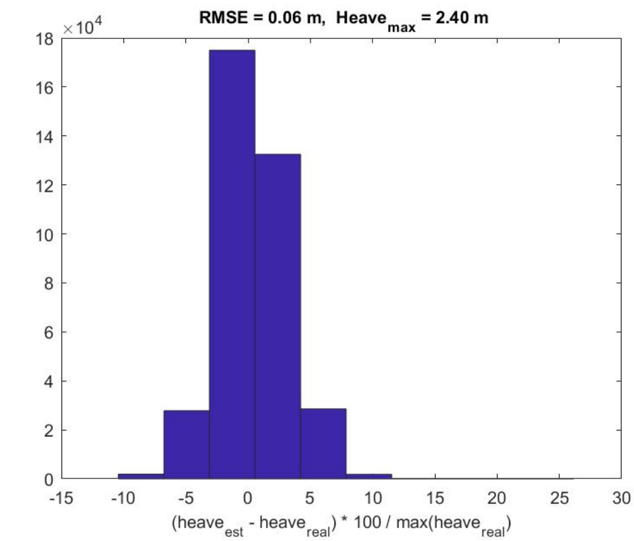

Fig. 12: the relative error and RMSE for the second scenario.

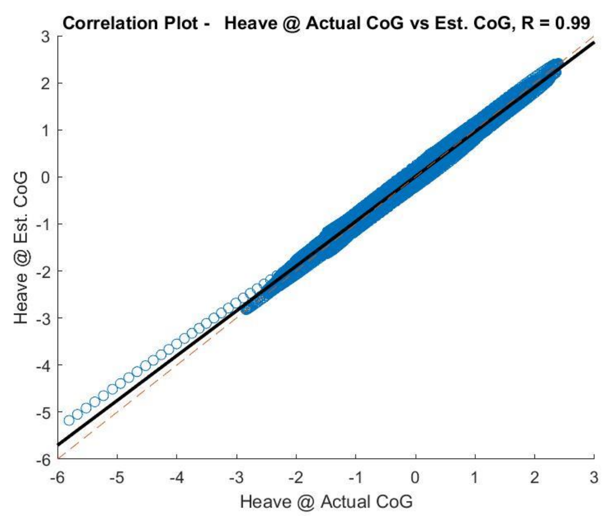

Fig. 13: Correlation between calculated heave using actual distances and the estimations for the second scenario.

Third scenario: IMU at the maximum lateral distance and further longitudinal position

For the third scenario, it was assumed that another (second) simulated iHeave was positioned at the maximum lateral distance which is equal of the half of the vessel's beam (the same as the second scenario) and a longer longitudinal position compared to the original. The calculated heave using the estimated distances was compared with the actual value in Fig. 14 and the error is presented in Fig. 15. While the error in the lateral and vertical distance is less than the scenario one, the RMSE of the derived body heave is the same. Fig. 16 illustrates the correlation between the calculated heave using the actual values and the estimation distances.

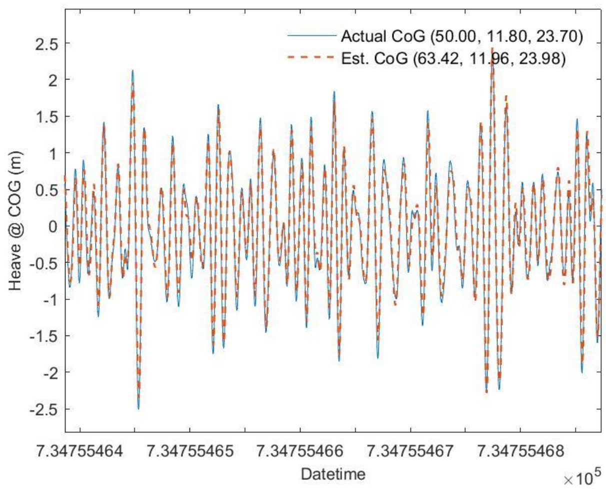

Fig. 14: comparison of calculated heave at actual/reported distance from CoG with the estimated values for the third scenario.

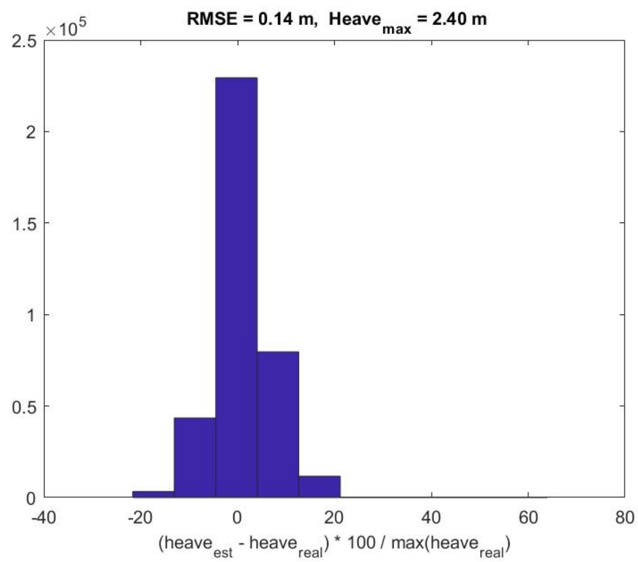

Fig. 15: the relative error and RMSE for the third scenario.

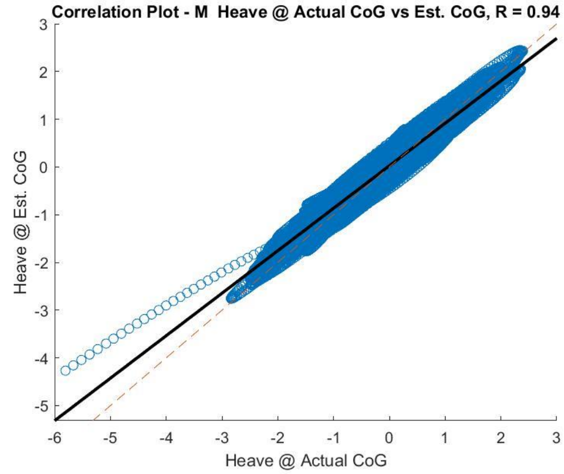

Fig. 16: Correlation between calculated heave using actual distances and the estimations for the third scenario.

EFFECT OF SAMPLING FREQUENCY AND FILTERING ON COG ESTIMATION

Sampling frequency and filtering are other factors which could have impact on the accuracy of the estimation. Initial investigation proved that filtering and down-sampling of the raw recorded data to 1 Hz could improve the estimations. Therefore, it is expected applying some averaging/smoothing functions could be necessary in preprocessing stage. New set of measurements for a laden bulk carrier along the Australian west and south coasts was used (Hibbert, et al., 2013) to investigate the effect of filtering and reducing the sampling frequency on the estimation. The distance between the iHeave located on the vessel's bridge to CoG is presented in Table 4. The sampling frequency of unprocessed data was 20 Hz. As shown in Table 4, the algorithm was unable to estimate the distance between iHeave and CoG with using the unprocessed data. Later, A noise filtering function has applied to the data and the sampling frequency reduced to 1 Hz. The estimated distance between iHeave and CoG with using processed data is presented in Table 5 and compared with using pure raw data. As shown in Table 5, both filtering and sampling frequency reduction improved the CoG estimations.

Table 4: Actual iHeave position vs the estimated position for bulk carrier transit between Port Hedland and Port Kembla with unprocessed data

	Actual position value (m)	Estimated value (m)
r_{v_x}	-114.53	0.35
r_{v_y}	-0.73	0.02
r_{v_z}	38.6	4.6

Table 5: Comparing the effect of down-sampling and noise filtering on the CoG estimation.

	Raw data	Sampling reduction	Filtering	Sampling reduction & filtering
Sampling frequency (Hz)	20	1	20	1
Filtering	No	No	Yes	Yes
Estimated r_{v_x}	0.35	-77.92	-110.69	-110.13
Estimated r_{v_y}	0.02	-0.96	-1.96	-0.87
Estimated r_{v_z}	4.6	33.05	32.34	34.04

Randomly, 100 positions were selected and the local motions at those points used to estimate the distance between the points and the CoG separately.
The considered range for longitudinal direction (x) was between -86 m and 86 m, -12 to 12 m for lateral (y), and -10 to 40 m for the vertical (z) distances. Fig. 17 ~ 19 show the correlation between the estimated and actual longitudinal, lateral, and vertical locations respectively. Fig. 20 illustrates the actual distance and the estimated values for those 100 points. The filled marker with thick line shows the best estimation (with minimum error). As can be seen the level of accuracy is dependent on the measurement location.

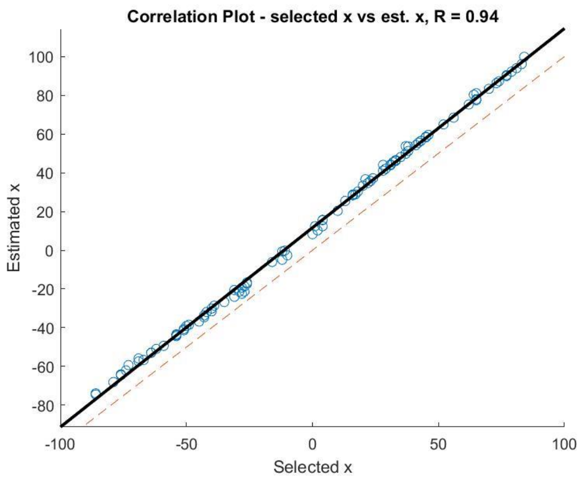

Fig. 17: correlation between selected longitudinal distance and estimated values

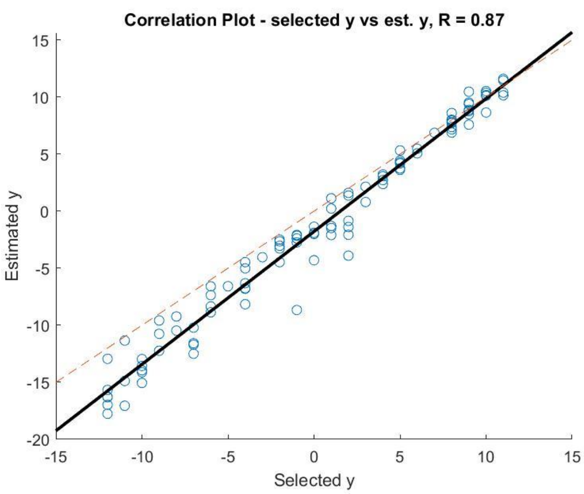

Fig. 18: correlation between selected lateral distance and estimated values

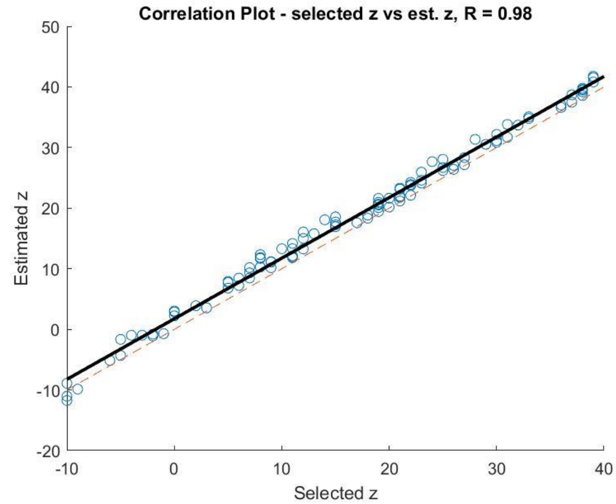

Fig. 19: correlation between selected vertical distance and estimated values

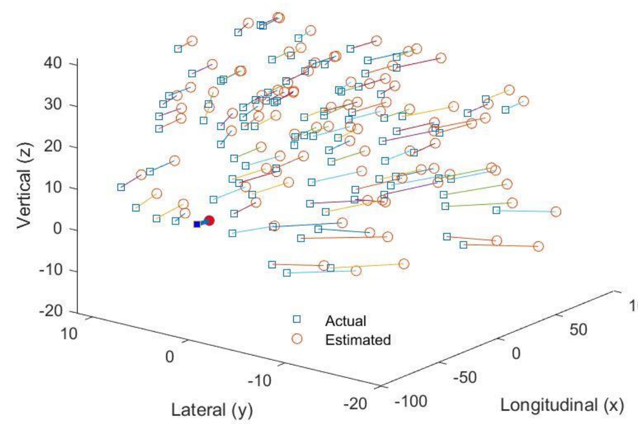

Fig. 20:Actual distance between the selected points and CoG vs estimated distance; thick line shows the best estimation with the minimum error.

CONCLUSION

Estimations of the centre of gravity position can be used for purposes such as sanity check on the reported stability values such as KG and LCG, or for when those parameters are not available. It can also be used to decrease the dependency on input time-dependent variables for autonomous shipping and support systems.
This report briefly explains the investigated method for estimating the distance between the centre of gravity (CoG) and the measurement point. According to the investigation, the proposed technique can accurately decompose the local motions (at the IMU position) and calculate the body heave motion (at CoG). Moreover, the technique can estimate the distance between the measured point and CoG; however, in general the level of accuracy is dependent on measuring point, filtering and sampling frequency.

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